Tuning between Exponential Functions and Zones for Membership Functions Selection in Voronoi-based Zoning for Handwritten Character Recognition

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Abstract — **In Handwritten Character Recognition, zoning is rigtly considered as one of the most effective feature extraction techniques. In the past, many zoning methods have been proposed, based on static and dynamic zoning design strategies. Notwithstanding, little attention has been paid so far to the role of function-zone membership functions, that define the way in which a feature influences different zones of the pattern. In this paper the effectiveness of membership functions for zoning-based classification is investigated. For the purpose, a useful representation of zoning methods based on Voronoi Diagram is adopted and several membership functions are considered, according to abstract–, ranked- and measurement-levels strategies. Furthermore, a new class of membership functions with adaptive capabilities is introduced and a real-coded genetic algorithm is proposed to determine both the optimal zoning and the adaptive membership functions most profitable for a given classification problem. The experimental tests, carried out in the field of handwritten digit recognition, show the superiority of adaptive membership functions compared to traditional functions, whatever zoning method is used.**

Keywords-component; formatting; Handwritten Character Recognition, Feature Extraction, Zoning, Membership Functions, Voronoi Diagrams

I. INTRODUCTION

In the field of handwritten character recognition, zoning is considered one of the most effective feature extraction technique able to handle handwritten pattern variability, due to different writing styles and personal changeability in writing. Strictly speaking, given a pattern image B, a zoning $Z_M=\{z_1, z_2, ..., z_M\}$ of B is a partition of B into M sub-images, named zones, each one providing information related to a specific part of the pattern [1].

Traditional approaches use static zoning methods, in which zoning design is obtained by standard grids that are superimposed on pattern images. In this case, no a-priori information on feature distribution is used for defining the zoning method. More recently, dynamic zoning techniques have been proposed, in which zoning design is considered as an optimization problem and the optimal zoning method is found as the zoning which maximizes the classification performance, estimated by a well-defined cost function associated to the classification task [2, 3]. For this purpose, Voronoi Diagrams have been recently proposed for zoning description, since they provide, given a set of points (named *Voronoi points*) in continuous space, a means of partitioning the space into sub-regions (named *zones*) according to proximity relationships among the set of points [4].

Unfortunately, although zoning is largely adopted and its effectiveness is widely demonstrated, aspects related to the choice of feature-zone membership functions have not been addressed yet. Notwithstanding, membership functions play a crucial role in exploiting the potential of a zoning method since they should be able to model the way in which the features detected in a pattern influence different zones. Thus, when zoning is used, the choice of a membership function needs specific attention.

Starting from this consideration, this paper investigates the effectiveness of traditional membership functions and introduces a new membership function with adaptive capabilities. Moreover, the paper presents a real-coded genetic algorithm for determining both the optimal zoning method, based on Voronoi Diagram, and the adaptive membership function most profitable for a given classification problem.

The experimental tests have been carried out in the field of handwritten digit recognition, using datasets extracted form the CEDAR database. The results show that the effectiveness of a zoning method strongly depends on the membership function considered. In addition, they demonstrate that adaptive membership functions are superior to traditional functions, whatever zoning method is used.

The paper is organized as follows. Section II introduces the zoning methods and the description technique by Voronoi Diagram. Zoning-based classification along with the problem membership functions selection is focused in Section III. Section IV presents the new class of adaptive membership functions and its application for classification using a real-coded genetic algorithm. Section V shows the experimental results, carried out on handwritten digits extracted by the CEDAR database. The conclusion of the paper is discussed in Section VI.

II. ZONING DESCRIPTION BY VORONOI DIAGRAM

Voronoi Diagram is a widespread technique of computational geometry that has been applied to several fields, ranging from biology to chemistry, from VLSI chip design to medicine, form physics to cartography [4]. Strictly speaking, given a set of a finite number of M distinct points $p_1,p_2,\dots,p_M$ in the Euclidean plane, the Voronoi Diagram is the partition of the plane into M zones $z_1,z_2,\dots,z_M$ that reflects proximity relationships among the set of points. In other words, each point p_i determines a region z_i that is the

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locus of points which are closer to p_i than to any other point of the set, according to the Euclidean distance [4].

Voronoi Diagrams have been recently used for zoning description [3, 5]. Static zoning methods are defined without using a-priori information on feature distributions. They are designed according to personal experience of the designer and experimental tests. As described in the literature, static zoning generally use regular partitioning criteria of the pattern image [1, 2]. Typical static zonings divide a $D_x \times D_y$ pattern image (D_x: image width, D_y: image height} into r × s identical rectangles [2]. A regular r × s zoning is represented by a Voronoi Diagram based on the set of M (M= r·s) Voronoi points $P=\{p_1, p_2, ..., p_M\}$, with

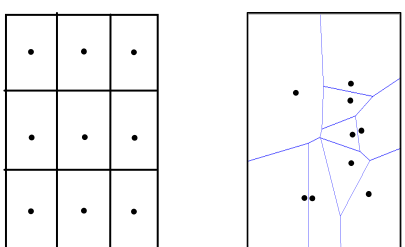
$$p_i = (px_i ,py_i) \qquad i=1,2,\ldots,M \tag{1}$$

where:

- $px_i=(1/2+k_x) \cdot D_x / s \quad , \; k_x =0,1,2,...,s-1$
- $py_i=(1/2+k_y) \cdot D_y / r \quad , \; k_y =0,1,2,...,r-1.$

being $i = 1 + k_x \cdot r + k_y$.

Dynamic zoning methods are defined on the basis of a-priori information on feature distributions, according to a specific optimality criterion [3]. Unlike static methods, that use standard partitioning criteria of the pattern image, dynamic zoning methods consider zoning design as an optimization problem and the optimal zoning $Z^*_M=\{z^*_1, z^*_2, ..., z^*_M\}$ is found as the zoning for which the cost function $CF(Z_M)$ associated to classification is minimum [3, 5]. Also in this case, if a zoning method $Z_M=\{z_1, z_2, ..., z_M\}$ is described by the set of Voronoi points $P=\{p_1, p_2, ..., p_M\}$, the optimal zoning design can be re-formulated as the problem of finding the set of Voronoi points $P^*=\{p^*_1, p^*_2, ..., p^*_M\}$ so that the corresponding zoning method $Z^*_M=\{z^*_1, z^*_2, ..., z^*_M\}$ (i.e. Z^*_M is the Voronoi Diagram defined from the set P^*) leads to the minimum of the cost function associated to classification performance.



(a) Z_{3x3} (b) Z^*_9

Figure 1. Static and dynamic zoning methods.

Figures 1 show two zoning methods represented by Voronoi Diagrams (the Voronoi points in the figures are also reported). Figures 1a shows the static zoning method based on a 3x3 grid (Z_{3x3}), Figures 1b shows the dynamic zoning method of 9 zones (Z^*_9).

III. ZONING-BASED CLASSIFICATION

Let us consider the classification of a pattern x into one of the classes in $\Omega = \{C_1 ,C_2, ...,C_N\}$ by the features of the set $F=\{f_1, f_2,...,f_T\}$ and using a zoning method $Z_M=\{z_1, z_2, ..., z_M\}$. In this case x can be described by a matrix (MAT_x) of TxM elements, where each element $MAT_x(i,j)$ reports the influence of the features of type f_i (i=1,2,…,T) detected in x on zone z_j (j=1,2,…,M). In other words, let $f_i(1), f_i(2),\ldots, f_i(q),\ldots,f_i(Q_i)$ be the Q_i instances of f_i in x, it results:

$$MAT_x(i, j) = \sum_{q=1}^{Q_i} w_{i_q j} \tag{2}$$

where $w_{i_q j}$ is the degree of influence of the instance $f_i(q)$ on zone z_j.

In general, let $Z_M=\{z_1, z_2, ..., z_M\}$ be a zoning method, for each instance of f_i detected at point pf_i, the influence weights w_{ij} are determined on the basis of proximity conditions between the position of f_i and z_j, j=1,2,…M. More precisely, let

$$Z_M=\{z_1, z_2, ..., z_M\} \tag{3}$$

be a zoning method corresponding to the Voronoi points

$$P=\{p_1, p_2, ..., p_M\}, \tag{4}$$

where z_j is the Voronoi region corresponding to the Voronoi point p_j , j=1,2,…,M;

Furthermore, let q_i the point in which feature f_i is found and let

$$d_{ij}=dist(q_i, p_j) \tag{5}$$

be the Euclidean distance between q_i and p_j;

The *Ranked Index Sequence* (RIS_i) associated to the feature f_i and denoting the sequences of zones, ranked according to their proximity to q_i, is defined as follows:

$$RIS_i = < i_1, i_2, \ldots, i_m, i_{m+1},\ldots, i_M > \tag{6}$$

so that

- $i_m \in \{1,2,\ldots,M\}$, $\forall m=1,2,\ldots,M$;
- $i_{m_1} \neq i_{m_2}$, $\forall m_1,m_2=1,2,\ldots,M$, $m_1 \neq m_2$;

and for which it results

$$d_{i m_1} < d_{i m_2} \;,\; m_1 < m_2 \;,\; \forall m_1,m_2=1,2,\ldots,M \tag{7}$$

(it is also assumed that, if $d_{i m_1} = d_{i m_2}$, then i_{m_1} precedes i_{m_2}, if $m_1<m_2$).

Furthermore, let

$$Count_i(j) \tag{8}$$

be the function providing the position of the index j (i.e. concerning zone z_j) in the sequence RIS_i (i.e. $count_i(j)=m$ for $j=i_m$, according to eq.(6)).

Under these assumptions, the following feature-zone membership functions can be considered [6]:

1. **Abstract-level membership functions:**

Membership functions at abstract-level assign Boolean influence weights to the zones:

- *The Winner-takes-all (WTA)* membership function:
  - $w_{ij}=1$ *if* $count_i(j) =1$ (9a)
  - $w_{ij}=0$ *otherwise;* (9b)
- *The k-Nearest Zone (k-NZ)* membership function:
  - $w_{ij}=1$ *if* $count_i(j) \in \{1,2,\ldots,k\}$ (10a)
  - $w_{ij}=0$ *otherwise* (10b)

2. **Ranked-level membership functions:**

Membership functions at ranked-level assign integer influence weights to the zones:

- *The Ranked-based (R)* membership function:
  - $w_{ij}=M-m$ *if* $count_i(j) =m$ (11)

3. Measurement-level membership functions:

Membership functions at measurement-level assign real influence weights to the zones. Three measurement-level membership functions are here considered :

- *The Measurement-based* membership functions:

- Linear Weighting Model (LWM)

- $w_{ij}=1 / d_{ij}$ (12)

- Quadratic Weighting Model (QWM)

- $w_{ij}=1 / d_{ij}^{2}$ (13)

- Exponential Weighting Model (EWM)

- $w_{ij}=1 / e^{\lambda \cdot d_{ij}}$ (14)

In addition, in this paper, a new adaptive technique to membership function design is introduced. It starts from the consideration that pattern features are spatially distributed according to local characteristics. In other words there are parts of the patterns in which features are confined to a small area (stable regions), as well as regions in which features are much more spread over a large area (variable regions). Therefore, membership function could be able to adapt itself to the local characteristics of patterns. For this purpose the following *Adaptive Weighting Model* (AWM) is considered. In particular for the each zone z_j we define an adaptive function of the kind

$$w_{ij} = e^{-\lambda_j \cdot d_{ij}} . \qquad (15)$$

where λ_j is a parameter determining the falling rate of the weighting model of zone z_j.

The adaptive model allows to define a wide range of weighting functions, depending on the falling rate λ_j, j=1,2,…,M. In particular, as the value of λ_j decreases, the area of influence of feature f_i augments and involves more and more zones of Z_M. Figure 2 shows the adaptive weighting models for different values of λ_j. It is worth noting that AWM works as a traditional WTA strategy, for λ_j=10; i.e. in this case feature f_i has influence (with $w_{ij}=1$) only on the zone z_j in which it is has been found. Conversely, when λ_j=0, f_i has the same influence (with $w_{ij}=1$) on all zones, no matter where f_i has been found.

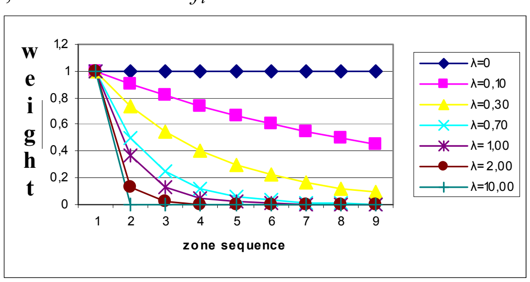


Figure 2. Adaptive Exponential Model (eq. 15).

IV. Adaptive Membership Functions for Classification by Voronoi Diagram

As stated in the previous sections, dynamic zoning design can be considered as an optimization problem. Of course, whatever zoning Z_M={z_1, z_2, ..., z_M} is considered, a fundamental role for the classification aims is also played by the feature-zone membership functions F_M={λ_1, λ_2, ..., λ_M}, where λ_j is the exponential value of the adaptive weighting model associated to z_j. Therefore, in this paper, the following cost function is used, which depends on both zoning method (Z_M) and membership function (F_M):

$$CF(Z_M, F_M) = \eta \cdot Err(Z_M, F_M) + Rej(Z_M, F_M) \qquad (16)$$

where [5, 21]:

- $Err(Z_M, F_M)$ is the error rate (estimated on the learning set);
- $Rej(Z_M, F_M)$ is the rejection rate (estimated on the learning set);
- the coefficient η is the cost value associated to the treatment of an error with respect to a rejection.

Moreover, since Voronoi Diagram is used for zoning description and the *Adaptive Weighting Model* is proposed as membership function, the problem of optimal zoning design becomes:

Find the sets

- {p^*_1, p^*_2, ..., p^*_M} *(Voronoi points)*
- {λ^*_1, λ^*_2, ..., λ^*_M} *(falling values)*

so that:

$$CF(Z^*_M, F^*_M) = \min_{\{ZM, FM\}} CF(Z_M, F_M) \qquad (17)$$

with:

- Z^*_M ={z^*_1, z^*_2,…, z^*_M} , z^*_j being the Voronoi region corresponding to p^*_j, ∀j=1,2,…,M ;
- Z_M ={z_1, z_2,…, z_M} , z_j being the Voronoi region corresponding to p_j , ∀j=1,2,…,M .

and

- F^*_M ={ λ^*_1, λ^*_2,…, λ^*_M} , λ^*_j being the falling value of the adaptive weighing model associated to the zone z^*_j , ∀j=1,2,…,M ;
- F_M ={λ_1, λ_2,…, λ_M}, λ_j being the falling value of the adaptive weighing model associated to the zone z_j , ∀j=1,2,…,M .

In order to solve the optimization problem (17), a genetic algorithm is proposed for the design of the adaptive membership function together with the optimal zoning [7].

The initial population Pop={Φ_1, Φ_2, ...,Φ_t, ... ,Φ_{NPop}} for the genetic algorithm is created by generating N_{pop} random individuals (N_{pop} even). Each individual is a vector

$$\Phi_i = \left\langle \begin{bmatrix} p_1 \\ \lambda_1 \end{bmatrix}, \begin{bmatrix} p_2 \\ \lambda_2 \end{bmatrix}, \dots, \begin{bmatrix} p_j \\ \lambda_j \end{bmatrix}, \dots, \begin{bmatrix} p_M \\ \lambda_M \end{bmatrix} \right\rangle \qquad (18)$$

where each element

$$\begin{bmatrix} p_j \\ \lambda_j \end{bmatrix} \qquad (19)$$

consists of:

- p_j: a point defined as $p_j=(x_j,y_j)$, that corresponds to the Voronoi point of the zone z_j of Z_M={z_1, z_2, ..., z_M};
- λ_j: a falling value that defines the adaptive weighting model for the zone z_j.

Consequently, the fitness value of the individual (eq. (18)) is taken as the classification cost $CF(Z_M, F_M)$, obtained by eq. (17), where:

- Z_M={z_1, z_2, ..., z_M} is the Voronoi Diagram, being z_j the Voronoi region corresponding to p_j, ∀j=1,2,…,M.
- F_M ={λ_1, λ_2,…, λ_M}, is the set of adaptive membership functions, being λ_j the falling value of

the adaptive weighing model associated to the zone z_j, $\forall j=1,2,\ldots,M$.

From the initial - population, the following genetic operations are used to generate the new populations of individuals [7]:

a) Individual Selection: $N_{pop}/2$ random pairs of individuals are selected for crossover, according to a roulette-wheel strategy.

b) Crossover: arithmetic crossover is used to combine information from diverse individuals. Let

$$\left\langle \begin{bmatrix} p^a{}_1 \\ \lambda^a{}_1 \end{bmatrix}, \begin{bmatrix} p^a{}_2 \\ \lambda^a{}_2 \end{bmatrix}, \ldots, \begin{bmatrix} p^a{}_j \\ \lambda^a{}_j \end{bmatrix}, \ldots, \begin{bmatrix} p^a{}_M \\ \lambda^a{}_M \end{bmatrix} \right\rangle \qquad (20a')$$

and

$$\left\langle \begin{bmatrix} p^b{}_1 \\ \lambda^b{}_1 \end{bmatrix}, \begin{bmatrix} p^b{}_2 \\ \lambda^b{}_2 \end{bmatrix}, \ldots, \begin{bmatrix} p^b{}_j \\ \lambda^b{}_j \end{bmatrix}, \ldots, \begin{bmatrix} p^b{}_M \\ \lambda^b{}_M \end{bmatrix} \right\rangle \qquad (20a'')$$

be two individuals selected for crossover, the two offspring individuals

$$\left\langle \begin{bmatrix} \tilde{p}^a{}_1 \\ \tilde{\lambda}^a{}_1 \end{bmatrix}, \begin{bmatrix} \tilde{p}^a{}_2 \\ \tilde{\lambda}^a{}_2 \end{bmatrix}, \ldots, \begin{bmatrix} \tilde{p}^a{}_j \\ \tilde{\lambda}^a{}_j \end{bmatrix}, \ldots, \begin{bmatrix} \tilde{p}^a{}_M \\ \tilde{\lambda}^a{}_M \end{bmatrix} \right\rangle \qquad (21b')$$

and

$$\left\langle \begin{bmatrix} \tilde{p}^b{}_1 \\ \tilde{\lambda}^b{}_1 \end{bmatrix}, \begin{bmatrix} \tilde{p}^b{}_2 \\ \tilde{\lambda}^b{}_2 \end{bmatrix}, \ldots, \begin{bmatrix} \tilde{p}^b{}_j \\ \tilde{\lambda}^b{}_j \end{bmatrix}, \ldots, \begin{bmatrix} \tilde{p}^b{}_M \\ \tilde{\lambda}^b{}_M \end{bmatrix} \right\rangle \qquad (21b'')$$

of the next generation are obtained as linear combination of the parent individuals, according to the random values $\alpha, \beta \in [0,1]$:

$$\tilde{p}^a{}_s = \alpha \cdot p^a{}_s + (1-\alpha) \cdot p^b{}_s; \qquad (22a)$$
$$\tilde{p}^b{}_s = \alpha \cdot p^b{}_s + (1-\alpha) \cdot p^a{}_s \qquad (22b)$$

and

$$\tilde{\lambda}^b{}_s = \beta \cdot \lambda^b{}_s + (1-\beta) \cdot \lambda^a{}_s \qquad (22c)$$

c) Mutation: a non-uniform mutation operator has been used. Let us consider the individual Φ_t and an element (see eq. (19)) selected for mutation, according to a mutation probability Mut_prob. The non-uniform mutation changes $\begin{bmatrix} p_j \\ \lambda_j \end{bmatrix}$ in the new element $\begin{bmatrix} \tilde{p}_j \\ \tilde{\lambda}_j \end{bmatrix}$ (being $\tilde{p}_j = (\tilde{x}_j, \tilde{y}_j)$) that is defined as follows:

c.1) Concerning $\tilde{p}_j = (\tilde{x}_j, \tilde{y}_j)$, it results (Fig. 3):

$$\begin{cases} \tilde{x}_j = x_j + \delta \cdot \cos(\varphi) \\ \tilde{y}_j = y_j + \delta \cdot \sin(\varphi) \end{cases} \qquad (28)$$

where:

- φ is a random value generated according to a uniform distribution, $\varphi \in [0, 2\pi[$;
- δ is a displacement determined according to the following equation:

$$\delta = \delta\_displ \cdot \left(1 - \nu^{\left[1 - \frac{iter}{N^{iter}}\right]^b} \right) \qquad (29)$$

being:

- ○ ν a random value generated in the range [0, 1], according to a uniform distribution;
- ○ δ_displ the maximum displacement allowed;
- ○ b a parameter determining the degree of non-uniformity;
- ○ *iter* the counter of the generations performed;
- ○ N^{iter} the maximum number of generations.

It is worth noting that eq. (29) causes the operator to search the space almost uniformly initially, when *iter* is small, and locally in later stages [7].

c.2) Similarly, concerning $\tilde{\lambda}_j$, we have:

$$\tilde{\lambda}_j = \lambda_j + (-1)^s \cdot \lambda\_displ \cdot \left[1 - \eta^{\left[1 - \frac{iter}{N_{iter}}\right]^c} \right] \qquad (30)$$

where

- ○ s is a random Boolean value generated according to a equally-distributed probability function;
- ○ η is a random value generated in the range [0, 1], according to a uniform distribution;
- ○ λ_displ the maximum displacement allowed;
- ○ c a parameter determining the degree of non-uniformity;

and where, also in this case, *iter* denotes the counter of the generations performed and N^{iter} denotes the maximum number of generations.

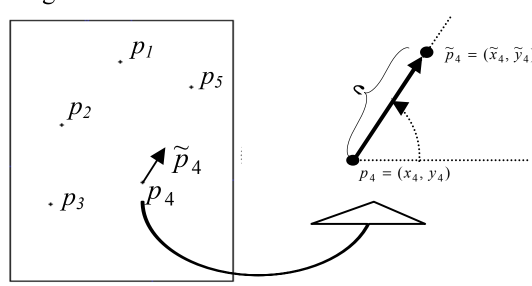


Figure 3. The Mutation Operator

- Elitist Strategy: from the N_{pop} individuals generated by the above operations, one individual is randomly removed and the individual with the minimum cost in the previous population is added to the current population.

Steps from (a) to (d) are repeated until N^{iter} successive populations of individuals are generated. When the process stops, the optimal zoning is obtained by the best individual of the last-generated population.

V. EXPERIMENTAL RESULTS

The experiments have been carried out using the set of handwritten numeral digits $\Omega_1=\{0,1,2,3,4,5,6,7,8,9\}$ extracted from the CEDAR database (BR directory) [8]. After normalization of each the pattern image to a size of 72x54 pixels, the skeleton of the pattern is derived through the *Safe Point Thinning Algorithm* [9]. Successively, the feature set $F=\{f_1,...,f_9\}$ is considered for pattern description, where (see [10] for more details): f_1 - *holes*; f_2 - *vertical-up cavities*; f_3 - *vertical-down cavities*; f_4 - *horizontal-right cavities*; f_5 - *horizontal-left cavities*; f_6 - *vertical-up end-points;* f_7 - *vertical-down end-points;* f_8 - *horizontal-right end-points;* f_9 - *horizontal-left end-points*.

In order to pre-estimate the most profitable parameter values for the Genetic Algorithm, some preliminary pilot tests have been conducted. According to previous studies in the literature [5, 7], the following parameter values have been considered: $N_{Pop}=10$; $N^{iter}=300$; *Mut_prob*=0.35; δ*_displ*=5 ; *b*=1.0; λ*_displ*=0.5, *c*=3.0. Figures 4 and 5 show the result obtained for the specific case of M=9, when handwritten digits are considered. Figure 4 shows the optimal zoning Z^*_9 , Figure 5 shows the set of optimal membership functions F^*_9.

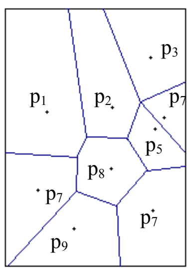


Figure 4. The Optimal Zoning Z^*_9

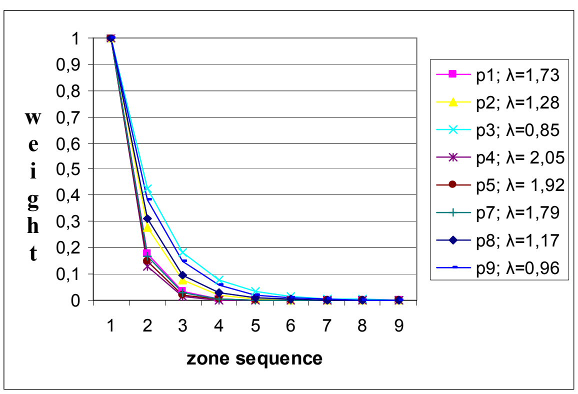


Figure 5. The Optimal Membership Function for F^*_9

Table I reports the experimental results by comparing the recognition rate of different membership functions on static and dynamic zoning methods. The results have been obtained according to 1-NN classifier and a k-fold cross-validation technique (k=10). The results show that dynamic zoning methods outperforms static methods, whatever number of zones (M) and Membership Function (F) is considered. In addition, the results demonstrate that AWM is always superior to other membership functions, whatever zoning is used. More precisely, the best results are achieved when AWM is used and the pattern image is dynamically partitioned into M=9 zones. In this case, the recognition rate is equal to 92%. The improvement with respect to a static zoning method (with M=9 zones) is equal to 14%. The improvement with respect to non-adaptive membership functions is up to 77%.

Table I. Experimental Results: Recognition Rate (η=5)

M	Zoning	*Performance*							
		Abstract			Ranked	Measurement			Adaptive
		WTA	2NZ	3NZ	R	LWM	QWM	EWM	AWM
4	Z_{2x2}	0.54	0.49	0.45	0.45	0.40	0.42	0.49	0.57
	Z^*_4	0.79	0.75	0.60	0.68	0.53	0.56	0.81	0.84
6	Z_{3x2}	0.74	0.70	0.62	0.50	0.44	0.50	0.73	0.79
	Z^*_6	0.87	0.84	0.75	0.62	0.52	0.57	0.87	0.89
9	Z_{3x3}	0.80	0.73	0.69	0.46	0.47	0.51	0.78	0.81
	Z^*_9	0.87	0.84	0.79	0.53	0.52	0.58	0.89	0.92
16	Z_{4x4}	0.82	0.79	0.77	0.44	0.49	0.47	0.80	0.81
	Z^*_{16}	0.88	0.85	0.84	0.54	0.54	0.63	0.90	0.91
25	Z_{5x5}	0.81	0.79	0.76	0.47	0.46	0.49	0.73	0.85
	Z^*_{25}	0.89	0.87	0.85	0.55	0.57	0.65	0.90	0.91

VI. CONCLUSION

This paper addresses the problem of membership function selection for zoning-based classification. Traditional membership functions, based non-adaptive strategies have been evaluated and compared with and a new adaptive membership function. The experimental results, carried out in the field of handwritten numeral recognition, demonstrate that the adaptive membership function leads to the best classification results, when compared to traditional membership functions.

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